JOURNAL OF AIRCRAFT
Vol. 43, No. 2, March–April 2006

Nozzle-Orientation Effects and Nonlinear Interactions Between Twin Jets of Complex Geometry

Rahul Joshi,* Praveen Panickar,† K. Srinivasan,‡ and Ganesh Raman§
Illinois Institute of Technology, Chicago, Illinois 60616

In this paper we focus on understanding the behavior of twin nozzles of complex geometry in various yaw orientations. To the best of our knowledge there are no published studies addressing the effects of nozzle orientation on the coupling of twin jets of complex exit geometry. We study the behavior of 1) uniform-exit rectangular nozzles, 2) single-beveled nozzles in a codirected configuration, and 3) single-beveled nozzles in a contradirected configuration. Experiments were carried out at fully expanded Mach numbers ranging from 1.28 to 1.72. Bevel angles of 10 and 30 deg were considered, and microphones located at the nozzle exit plane quantified the coupling using both linear and nonlinear spectral-analysis methods. Nonlinear characteristics were quantified using the nonlinear interaction density metric with a cross-bicoherence cut-off threshold of 0.4. The following interesting results emerged from this study: 1) When nozzles having uniform rectangular exits are yawed, the sound-pressure levels in the internozzle region reduce as the yaw angles are increased, and, at a very high yaw angle, the symmetric coupling regime that existed at the high fully expanded Mach number range (without yaw) is replaced by an antisymmetric coupling regime in the same range. 2) Geometrically similar exits from uniform-exit rectangular nozzles and beveled nozzles in the contradirected configuration showed similar characteristics when studied using linear techniques. However, they revealed information that was hitherto unknown when studied using nonlinear spectral-analysis techniques. It is believed that the results presented in this paper will provide benchmark data to those simulating/designing complex-geometry nozzle systems.

Nomenclature

$B.A.$ = bevel angle, deg
f = frequency, Hz
$I_{c,0.4}$ = interaction density above threshold cross-bicoherence values of 0.4
M_j = fully expanded jet Mach number
α = yaw angle, deg
β = total angle subtended by the nozzles, deg

I. Introduction

MODERN fighter aircraft use twin-thrust-vectoring nozzles of complex geometry; most notable are the F-22, which uses twin double-beveled nozzles, and the F117 and the B-2, both of which use distributed exhaust systems. It has been documented[1–5] that jets from twin nozzles can interact, resulting in completely different acoustic properties of the flow as compared to the individual jets. Twin-jet coupling can accelerate fatigue failure and has caused nozzle or tail-plane damage in aircraft such as the B1-B. It can also result in significant unsteady pressures in the internozzle region, which may cause damage to advanced composite material.

Past studies in this field have concentrated on single jets and twin configurations of simple geometry. There is a wealth of information on twin jets of simple geometries including theoretical studies such as instability calculations.[6] It has been only recently that complex-geometry nozzles have begun receiving attention,[7–9] mainly because of the thrust-vectoring advantages they offer. As far as the coupling phenomenon is concerned, although there is some literature on the coupling of jets from simple and complex geometries,[10,11] they mainly focus on the study of effects of exit geometry and parameters such as internozzle spacing and Mach number. There have also been studies that focused on suppressing the screech from single as well as twin jets.[12,13] To the best of our knowledge the only other studies regarding the effect of nozzle orientation on the interaction between twin jets were conducted by Zilz[14] and Zilz and Wlezien.[15] These studies were conducted on twin two-dimensional nozzles that had uniform-exit geometry. Their experiments included studying the effects of changing the axial orientation of the nozzles with respect to each other (which they termed as the "inward splay") as well as using sidewall cut-backs (which would essentially simulate complex exit geometries) on the overall sound-pressure level (OASPL) of the jets. Although data from cold laboratory jets are not directly applicable to aircraft, there is evidence in the literature (Berndt[2]) showing that the region where damage was detected in flight tests matched the region where high dynamic pressures were measured in twin cold laboratory jet experiments in the laboratory. We believe that these studies are important for aircraft applications because the nozzles of some aircraft are often set into yawing and pitching motions for thrust-vectoring purposes. Such maneuvers demand that the behavior of nozzles in various orientations be well understood.

II. Experimental Facility, Instrumentation, and Setup

All the experiments reported in this paper were carried out in the High Speed Jet Facility (HSJF) at the Illinois Institute of Technology, Chicago. The HSJF is a blow-down facility that provides a maximum momentary exit pressure ratio of 15.3 based on a nozzle-exit diameter of 1.0 in., which results in a fully expanded Mach number of 2.43. The air-supply system consists of a bank of four compressors, oil filters, air driers, storage tanks, a settling chamber, and a digital control system for controlling the stagnation pressure in the settling chamber.

The nozzles are attached to the settling chamber using flexible hosing. The jets from the nozzles exhaust into a foam-lined chamber. The chamber is anechoic for the frequencies of interest in this study. Each nozzle is mounted on a Velmex™ rotary table (used to

Received 24 August 2004; revision received 18 May 2005; accepted for publication 9 June 2005. Copyright © 2005 by the authors. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. Copies of this paper may be made for personal or internal use, on condition that the copier pay the $10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0021-8669/06 $10.00 in correspondence with the CCC.

*Graduate Student, Fluid Dynamics Research Center, Engineering 1 Building, 10 W. 32nd Street.

†Graduate Student, Fluid Dynamics Research Center, Engineering 1 Building, 10 W. 32nd Street. Member AIAA.

‡Visiting Assistant Professor, Fluid Dynamics Research Center, Engineering 1 Building, 10 W. 32nd Street; currently Department of Mechanical Engineering, IIT Madras, India. Senior Member AIAA.

§Associate Professor, Fluid Dynamics Research Center, Engineering 1 Building, 10 W. 32nd Street; raman@iit.edu. Associate Fellow AIAA.

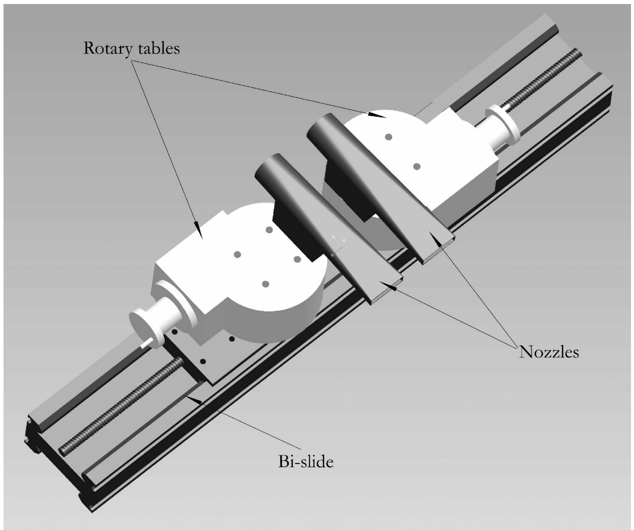


Fig. 1 Schematic of the experimental setup for studying twin-jet interactions under yawing motion. The yawing motion is simulated with the help of the rotary tables and the internozzle separation is maintained using the Bi-Slide™.

control the yaw angle), that is mounted on a Velmex Bi-Slide (used to maintain the internozzle separation in a way as to ensure that the nozzle tips touch each other during the run). Figure 1 shows a schematic of the experimental setup.

The yaw angle can be defined depending on the nozzle orientation and/or bevel angle (Fig. 2). For the present study, the nozzles are moved in such a way that the nozzle axes intersect each other, as shown in Fig. 2a. The included angle α subtended between the axes is termed the yaw angle. Another important parameter is the total angle subtended by the nozzle exits, as shown in Fig. 2b, during the yawing motion, referred to in this report as the total angle β. As seen in Fig. 2b, β depends on the twin-jet configuration (codirected or contradirected) and the bevel angle. In general, the angle β can be written as

$$\beta = 180 \pm 2 \times B.A. - \alpha \tag{1}$$

For a uniform-exit rectangular nozzle, $B.A. = 0$. In this equation, the + sign refers to the contradirected configuration and the − sign refers to the codirected configuration.

To change the yaw angle, the nozzles were moved apart, after which the angles were adjusted using the rotary tables, and then the internozzle separation was adjusted to make the nozzles touch each other. The procedure is repeated for all the yaw angles. The internozzle separation and the yaw-angle adjustment were computer controlled using LabVIEW 6i. It must be noted that during the yawing operation, the inner tips of the nozzles touched each other as shown in Fig. 2a.

Sound-pressure levels were measured using $\frac{1}{4}$ in. B&K 4939 microphones that were calibrated using a pistonphone. The experiments were carried out for fully expanded jet Mach numbers from 1.28 to 1.72.

III. Objectives

Our specific objectives in this study are

1) to identify different modes and the transition from one mode to the other through yawing of twin jets,

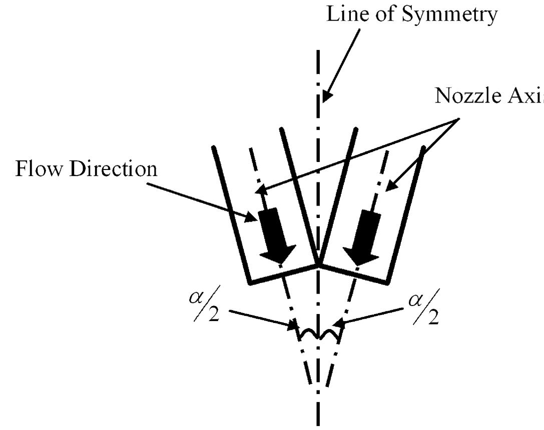
2) to better understand the nonlinear effects in the coupling phenomena through nonlinear spectral analysis, and

3) to provide an experimental database of nozzle-orientation effects on twin-jet coupling for those performing simulations and for nozzle designers.

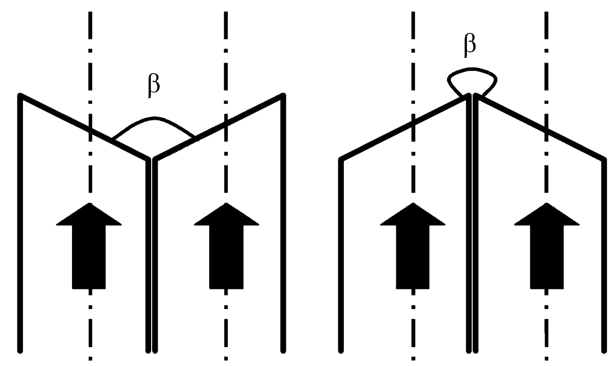
IV. Results and Discussion

Results are presented for the three twin-jet configurations considered, namely, uniform-exit nozzles, single-beveled ($B.A. = 30$ deg) nozzles in V configuration (codirected), and single-beveled ($B.A. = 10$ deg) nozzles in arrowhead configuration (contradirected). The choice of 10-deg beveled nozzles for the contradirected configuration was made based on the study by Panickar et al.[16] that showed no coupling for the $B.A. = 30$ deg single-beveled nozzles in the contradirected configuration. For each case studied, the effect of yaw angle on sound pressure level (SPL), screech frequency, and spanwise phase difference has been presented. The results of nonlinear spectral analysis have also been presented for a few selected cases.

Before moving on to the actual discussion, a few words on the error analysis and uncertainty in the data are warranted. For a few selected cases, the experimental data were repeated a number of times and the deviations in the values were obtained using statistical methods. Using these methods we were able to compute the maximum and minimum deviations in the sound-pressure levels, the phase difference, and the frequencies. It was found that these values changed depending on the operating conditions. The maximum error in the sound-pressure levels ranged from +3.9 to −2.6 dB depending on the operating condition, which was within an error of 2.5%. Similarly, for the phase difference, the uncertainties ranged from an upper limit of 10.7 deg to a lower limit of −11.5 deg. The frequency measurement was repeatable to a high degree of accuracy, and the error in the frequency was found to be lower than 2%. Error bars have been shown on a few key figures to visually illustrate the uncertainty levels.



a)



b)

Fig. 2 Definition sketch for yaw angle α and total angle β: a) Schematic of yawing operation. In this case the nozzles move in opposite directions with respect to the line of symmetry. Note that the inner tips of the nozzles touch each other during operation, which is the case at all the yaw angles. The yaw angle α is the sum of the angles made by each nozzle axis with the line of the symmetry. b) Total angle β. The value of β can be calculated according to Eq. (1) depending on the twin-jet configuration under consideration, codirected (shown on the left) or contradirected (shown on the right).

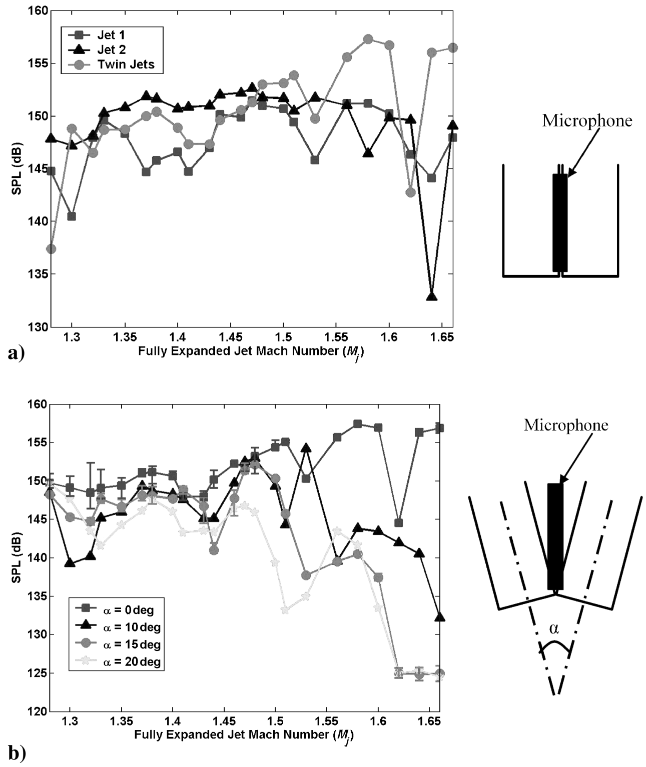


Fig. 3 SPL map over the fully expanded Mach number range of operation for uniform exit rectangular jets: a) Comparison between peak SPL levels of single and twin jets at a yaw angle of 0 deg. The schematic shown alongside shows the microphone location. The microphone location is kept the same for single- and twin-jet operation. b) Comparison between SPL levels for twin jets at various yaw angles, α. The microphone location is kept the same as for a).

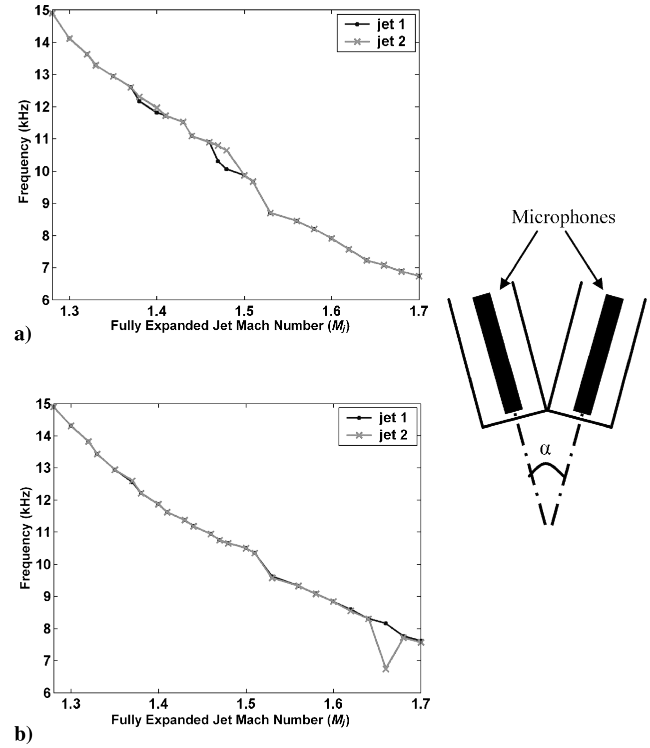


Fig. 4 Frequency map over the fully expanded Mach number range of operation for uniform exit rectangular jets. Microphone locations for the measurements are shown in the schematic alongside. Note that both jets operate at the same frequency for almost the entire range of fully expanded Mach numbers: a) Yaw angle α = 0 deg. b) Yaw angle α = 20 deg.

A. Nozzle-Orientation Effects

1. Uniform-Exit Rectangular Nozzles

Figure 3a shows the variation of tonal sound-pressure levels with Mach number for the individual- and twin-jet configuration with no yaw. Previous studies on nozzle-orientation effects on twin two-dimensional uniform-exit rectangular nozzles[14,15] used nozzles with aspect ratios of around 3 (as opposed to nozzles with an aspect ratio of 7 as used in this study). The maximum inward-splay angle used in those studies was 8.2 deg. This corresponds, for the purposes of our study, to a yaw angle of 16.2 deg. They[14,15] reported reduction in the OASPL up to 10 dB. Figure 3b shows the peak sound-pressure levels produced by the twin jets for yaw angles of 0 to 20 deg. At all yaw angles, the sound-pressure level was measured by a microphone located along the line of symmetry between the two nozzles as shown in the schematic alongside the figures. Note that one of the microphone locations was the same as that used in a previous study.[14,15] Our studies show very similar trends to the previous work, wherein we observe a reduction in the peak SPL with an increase in the yaw angles at virtually all the fully expanded Mach numbers.

An important parameter that needed to be verified was whether the jets were frequency locked (as done previously in Panickar et al.[16]). This was done by placing two microphones at the spanwise centers of the two nozzles and recording the screech frequency recorded by each microphone. In the present study, it was found that the jets were indeed frequency locked over almost the entire Mach-number range of operation. For instance, Figs. 4a and 4b show the frequency recorded by the individual microphones, located as shown in the schematic alongside, at yaw angles of 0 and 20 deg. As seen from the figures, both jets have the same screech frequency over the entire operating range, which proves that they are frequency locked.

Figure 5 shows the phase difference measured between the microphones located at the spanwise centers of the two nozzles (as

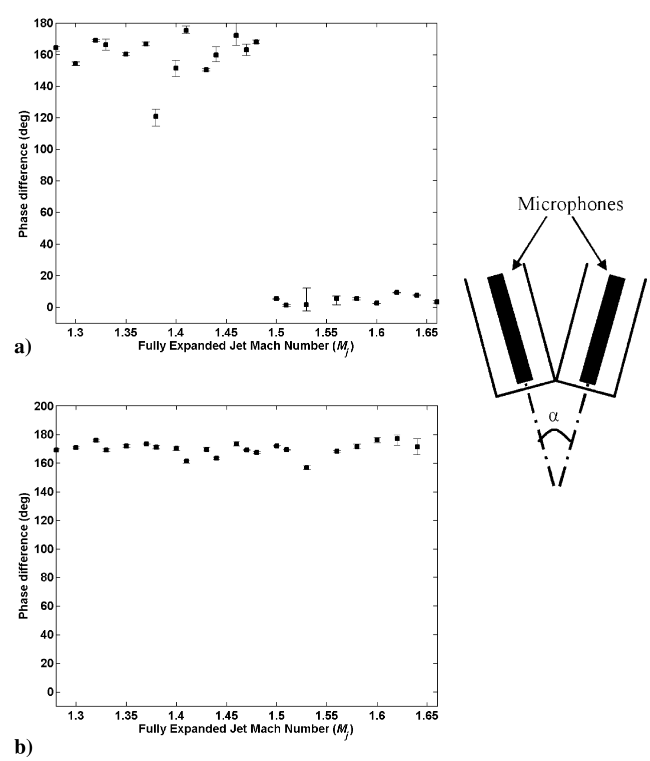


Fig. 5 Phase map over the fully expanded Mach-number range of operation for uniform-exit rectangular jets. Microphone locations are shown in the schematic alongside: a) Yaw angle α = 0 deg. The phase is close to antisymmetric at the low fully expanded Mach numbers and switches to symmetric at the higher fully expanded Mach numbers. b) Yaw angle α = 20 deg. Note that the phase is antisymmetric over the entire range of fully expanded Mach numbers.

shown in the schematic alongside) at a couple of different yaw angles. At 0-deg yaw angle (when the twin jets' axes are parallel), the twin jets exhibited antisymmetric coupling at the lower range of fully expanded Mach numbers and symmetric coupling in the higher range of Mach numbers (see Fig. 5a) in conformance with the results obtained by Raman and Taghavi.[10] As the yaw angle is increased, the symmetric coupling that was seen at the higher Mach-number range at smaller yaw angles disappears and is replaced by an antisymmetric coupling. Finally, at the higher yaw angles the jets are antisymmetrically coupled over the entire Mach-number range as can be seen in Fig. 5b, which is the phase map at a yaw angle of 20 deg. Recognizing that the phase information is meaningless without corresponding coherence values, the authors checked the coherence values of the phase data presented in Fig. 5 and confirmed that the coherence values were uniformly high (greater than 0.85).

2. Single-Beveled Nozzles (B.A. = 30 Deg) in Codirected Configuration

The 30-deg bevel angle, single-beveled-exit jets in the codirected (V-shaped configuration) show symmetric coupling at the lower fully expanded jet Mach numbers and antisymmetric coupling at the higher range of fully expanded jet Mach numbers, at a 0-deg yaw angle, as reported by Panickar et al.[16] However, when these jets are yawed at yaw angles of more than 10 deg, the coupling seems to vanish. Microphone measurements confirmed that the jets were no longer frequency locked and the coherence between the signals from the jets dropped to unacceptably low values (below 0.5 in some cases). In addition, the spectra of twin jets showed no screech tones present at higher yaw angles. We suspect that when the jets are vectored toward each other, screech is eliminated because of the destruction of the feedback loops.

In their previous work, Panickar et al.[16] and Srinivasan et al.[17] had tested 30-deg bevel angle, single-beveled nozzles in the contradirected configuration and had reported that the jets did not interact, linearly or nonlinearly, in this configuration. This fact, and the fact that the codirected jets did not frequency lock at higher yaw angles, led the authors to believe that a 30-deg bevel angle was perhaps too large for the purpose of studying the effects of yawing. Consequently, we considered two nozzles with a bevel angle of 10 deg for the next round of experiments.

3. Single-Beveled Nozzles (B.A. = 10 Deg) in Contradirected Configuration

The 10-deg bevel angle, single-beveled nozzles were examined in the contradirected configuration instead of the codirected configuration. This was because in the codirected configuration, any increase in yaw angle would vector the jets toward each other and cause each jet to disrupt the flow of the counterpart as was the situation described in Sec. IV.A.2. This situation would hardly arise in practical circumstances and hence was not considered for this round of experiments. The 10-deg bevel angle, single-beveled nozzles in arrowhead configuration showed very interesting coupling-mode switching behavior. Figure 6 shows the SPL map of the twin-jet configuration over the entire fully expanded Mach number range at a couple of the higher yaw angles. The frequency and phase variation for this configuration are discussed briefly. However, the data have been omitted from the paper in the interest of brevity.

For each 10-deg bevel angle case, it was verified that the jets remained frequency locked at the higher yaw angles. This behavior is in sharp contrast to that observed for the 30-deg bevel angle case. A study of the spanwise phase difference as a function of the yaw angle revealed that the jets were antisymmetrically coupled at the lower fully expanded jet Mach numbers and symmetrically coupled at the higher fully expanded jet Mach numbers. An important observation that surfaces from studying the phase map and the SPL map simultaneously is that SPL levels for the symmetrically coupled region tend to reduce as compared to the SPL levels for the antisymmetrically coupled regions. This is a finding that is hitherto unreported, which goes on to show that symmetric coupling does not always lead to an increase in the internozzle sound-pressure levels. Thus, an increase in sound-pressure levels is not enough to conclude the presence of symmetric coupling but rather is further necessary to validate the experimental data by two-point phase measurements before a final conclusion can be derived.

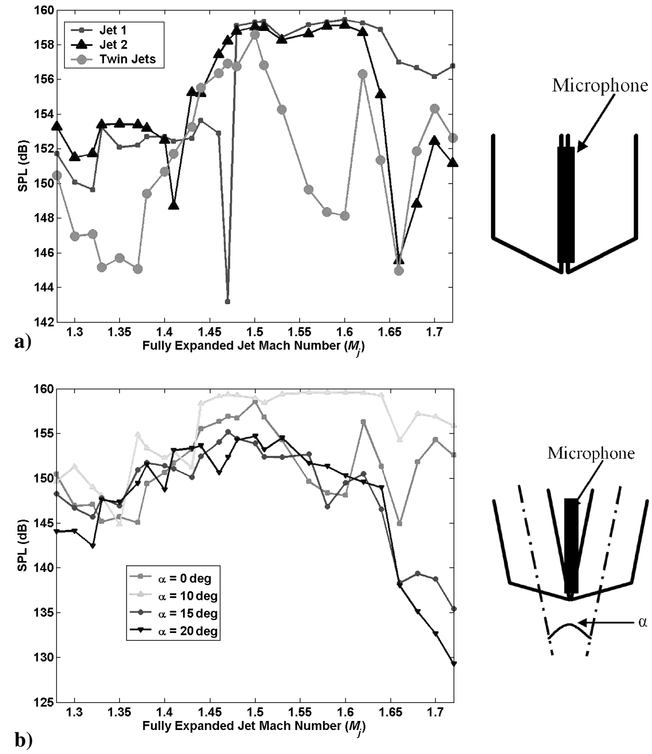


Fig. 6 SPL map over the fully expanded Mach-number range of operation for single-beveled jets ($B.A.$ = 10 deg): a) Comparison between SPL levels of single and twin jets at a yaw angle of 0 deg. The schematic shown alongside shows the microphone location. The microphone location is kept the same for single- and twin-jet operation. b) Comparison between SPL levels for twin jets at various yaw angles, α. The microphone location is kept the same as for a).

Another interesting observation that could be made was the fact that when the contradirected 10-deg bevel angle jets were yawed, the total angle β, as given by Eq. (1), reduces. When the yaw angle is 20 deg, the value of $\beta = 180$ deg. Conversely, for uniform-exit rectangular nozzles, at a 0 deg yaw angle, the total angle is also $\beta = 180$ deg. Thus, because the two exits are geometrically similar, linear methods yield the same qualitative results for identifying the coupling mode; that is, both the uniform-exit rectangular nozzles and the 10-deg bevel angle, single-beveled nozzles yawed at 20 deg and showed antisymmetric coupling characteristics at the lower range of Mach numbers and symmetric coupling at the higher range of Mach numbers. It would be interesting to use higher-order spectral methods to determine any kinematic variations between the two configurations on the basis of simple acoustic measurements.

B. Nonlinear Spectral Analysis

Nonlinear spectral-analysis techniques have the potential to yield answers to questions that have perplexed researchers for awhile now. In this paper, the results from nonlinear analysis techniques are presented using the cross-bicoherence spectrum. It can be shown using synthetic spectra that linear spectral analysis is inadequate for distinguishing between linearly superposed and quadratically modulated signals. This limitation can be overcome by using higher-order spectral-analysis techniques. Generally speaking, a linearly superposed signal can be expressed as the sum of sine waves and can be identified in the sum interaction region of the cross-bicoherence spectrum. Conversely, a modulated signal can typically be expressed as the product of sines which can in turn be simplified using trigonometric relations as the difference of sines and can be identified in the difference interaction region of the cross-bicoherence spectrum.

Nonlinear techniques for complex-geometry twin jets were first used in the paper by Srinivasan et al.,[17] and these analyses revealed hitherto unknown facts about jet interaction. Prior to this, Ponton and Seiner[18] and later, Walker and Thomas[19] had used nonlinear techniques in describing high-speed jet flows. In their studies, Srinivasan et al.[17] used cross-bicoherence to quantify the nonlinear activity in the twin jets, and they found that the nonlinearity in the jet interaction increased as the jets transitioned from the symmetric to the antisymmetric coupling mode. Moreover, the jets that appeared not to interact on the basis of linear-spectral analyses were found to be interacting nonlinearly.

1. Uniform-Exit Rectangular Jets

We performed nonlinear spectral analysis on two sets of data for the uniform-exit rectangular jets in twin configuration to bring out the role of nonlinear interactions during mode switching (that is, coupling transition from antisymmetric to symmetric). The cases have been chosen such that one of them (Mach number 1.47) corresponds to the mode-switching region and the other (Mach number 1.53) exhibits a purely antisymmetric coupling mode. Both the cases presented correspond to a yaw angle of 10 deg. The two sets are analyzed for their individual characteristics and then compared with each other.

a. Mach Number 1.47 Case. The spectra of the two microphone signals are shown in Fig. 7b. The linear spectra show a few nonharmonically related frequencies apart from the fundamental screech frequency and its harmonics. It may also be observed that few frequencies are dominant in only one of the microphone signals. Two views of the cross-bicoherence spectrum are shown in Figs. 7a and 7c. The cross-bicoherence spectrum shows a dominance of lineal interactions or straightly aligned interactions involving more than one frequency. These are characterized by the dots aligned along horizontal and vertical lines in Fig. 7b.

To gain further insights into the nature of the nonlinear interactions, the nonlinear interactions are tabulated in Table 1. In this table, the two frequencies participating in the sum or difference interactions as well as the cross bicoherence of the interaction are tabulated. Also tabulated is the resultant frequency of the interactions. These resultant frequencies enable us to interrelate the linear and nonlinear spectra. For example, a frequency f_1, found in the linear spectrum, could be the resultant frequency of a nonlinear interaction, indicated by the nonlinear spectrum. In this work, we attempt to make such interrelationships between the linear and nonlinear spectra.

This is done as follows:

1) The spectral peaks in the linear spectra are identified.

2) The closeness of these linear-spectral peaks to the resultant frequencies of nonlinear interactions is tabulated. If the resultant frequency of a nonlinear interaction is close to a linear-spectral peak within the frequency resolution of the calculations, it is assumed that both peaks actually indicate a distinct mode (frequency).

3) Because a certain mode can result from more than one interaction, all such interactions that result in a distinct mode are grouped together. For example, mode $2f_1$ could result from a sum interaction $f_1 + f_1$ or a difference interaction $3f_1 - f_1$. These two interactions are grouped together in this analysis.

The sum of the nonlinear coherences in a group is calculated, and this sum is considered representative of the extent of nonlinearity associated in the production of that mode. These numbers that quantify the nonlinearity are tabulated in Table 2.

Table 2 shows that the fundamental screech mode and its first harmonic top the list with high values of the total cross bicoherence. This is followed by a nonharmonically related frequency (6386 Hz), which ranks third. The second harmonic ranks fourth in this list. Based on this table, it is clear that the fundamental frequency and its harmonics are involved in a significant part of the nonlinearity in the spectra. However, this analysis clearly points out the key role played by nonharmonically related frequencies in generating other nonharmonic peaks in the spectra. There are equal numbers of nonharmonic frequencies in Table 2, substantiating their role in the spectral evolution. This table also identifies modes not resulting from nonlinear interactions (first three frequencies in Table 2).

Table 1 Cross-bicoherence values of nonlinear interactions for the Mach number 1.47 case

No.	f_1	f_2	CBC	Resultant frequency
1	32,387	−10,789	0.9722	21,598
2	10,763	10,821	0.9629	21,584
3	43,249	−10,789	0.9393	32,460
4	43,249	−32,399	0.9242	10,850
5	17,710	−10,789	0.8468	6,921
6	32,387	10,821	0.8316	43,208
7	21,624	10,821	0.7755	32,445
8	21,624	−10,789	0.7631	10,835
9	32,387	6,812	0.6916	39,199
10	32,485	−21,643	0.6537	10,842
11	32,485	−17,634	0.6358	14,851
12	43,249	−21,643	0.6216	21,606
13	17,613	10,821	0.6140	28,434
14	17,710	−6,877	0.5442	10,833
15	14,775	6,910	0.4930	21,685
16	21,624	6,812	0.4861	28,436
17	32,485	−25,554	0.4839	6,931
18	21,624	17,666	0.4740	39,290
19	21,624	−14,798	0.4740	6,826
20	21,624	−6,780	0.4689	14,844
21	17,613	14,831	0.4524	32,444
22	43,249	−36,408	0.4405	6,841
23	43,346	−14,798	0.4320	28,548
24	25,538	−10,691	0.4283	14,847
25	21,624	21,675	0.4136	43,299
26	25,538	−14,798	0.4043	10,740
27	10,763	6,812	0.4036	17,575
28	28,474	10,724	0.4031	39,198
29	32,387	14,733	0.3945	47,120
30	25,538	6,910	0.3842	32,448
31	28,474	−17,634	0.3745	10,840
32	43,249	−6,780	0.3696	36,469
33	10,665	8,768	0.3551	19,433
34	36,399	−14,798	0.3515	21,601
35	28,474	−6,877	0.3431	21,597
36	39,237	−32,399	0.3236	6,838
37	39,237	−17,634	0.3175	21,603
38	36,399	−25,554	0.3098	10,845
39	36,399	10,724	0.3080	47,123

Several nonharmonically related frequencies.
Screech frequency could be 10,791 Hz for one jet and slightly different for the other jet.
Note: f_2 with a minus sign indicates a difference interaction.

Table 2 Quantum of nonlinearities associated with frequencies identified in linear spectra

Spectral peak, Hz	Total CBC	Remarks
25,537	——	No prominent nonlinearity involved
36,328	——	No prominent nonlinearity involved
46,070	——	No prominent nonlinearity involved
10,791	3.974	Fundamental screech frequency
21,582	3.569	First harmonic
6,836	2.568	Nonharmonic frequency
32,373	2.551	Second harmonic
39,209	1.569	Nonharmonic
14,746	1.533	Nonharmonic
43,164	1.245	Third harmonic
28,418	1.1	Nonharmonic
17,627	0.404	Nonharmonic

b. Mach Number 1.53 Case. The spectra of the two microphone signals are shown in Fig. 8b, and two views of the cross-bicoherence spectrum are shown in Figs. 8a and 8c. Although the nonlinear interactions are lineal, the extent is much smaller compared to the Mach number 1.47 case. The entire participation is by the fundamental frequency and its harmonics. The nonharmonically related peak has an insignificant contribution in the nonlinear dynamics of coupling (see Tables 3 and 4).

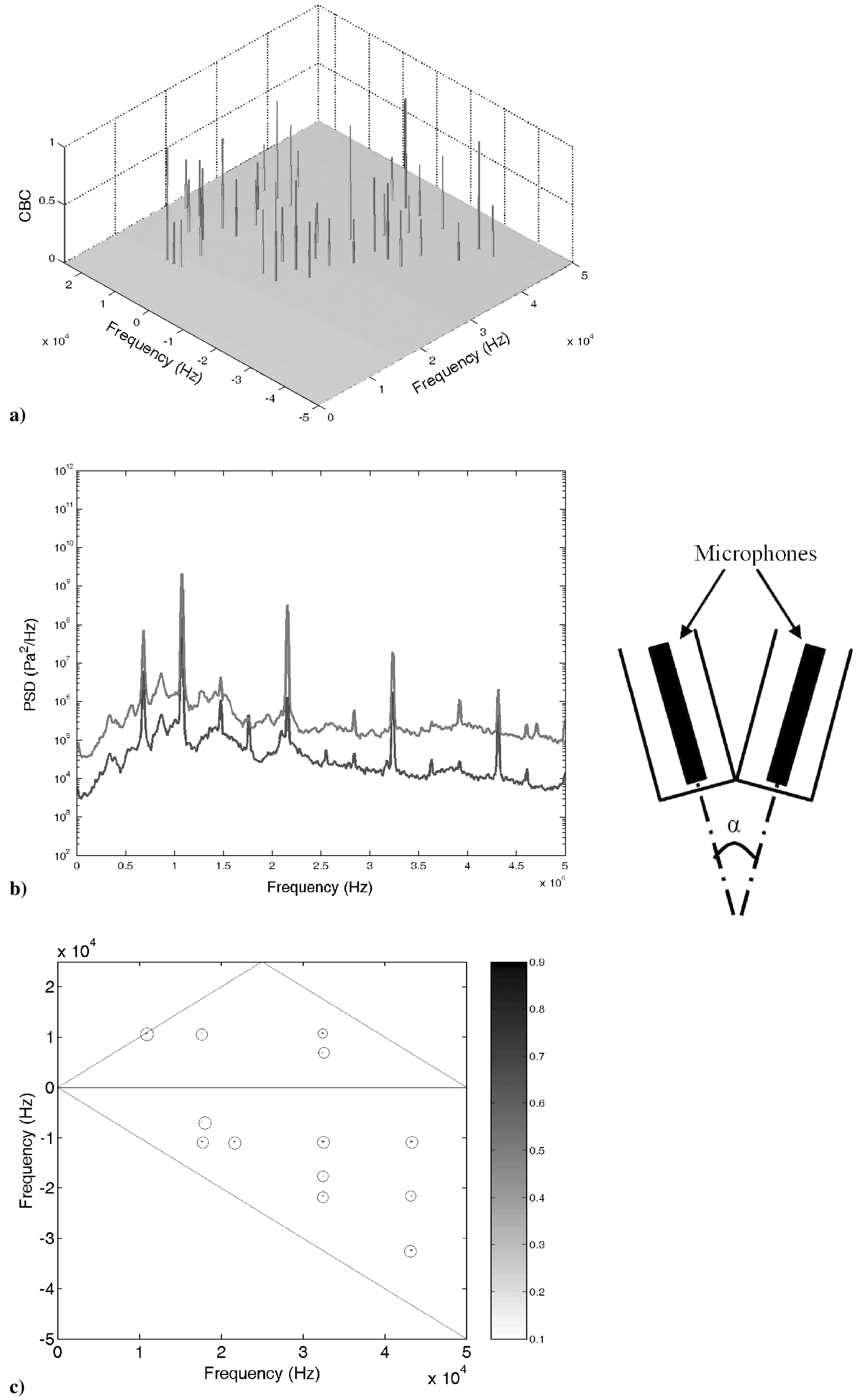


Fig. 7 Linear and CBC spectra for uniform-exit rectangular jet configurations at Mach number 1.47 as measured by microphones, located at the spanwise center of the nozzles as shown in the schematic: a) Three-dimensional view of the cross-bicoherence spectra between the two microphone signals at $M_j = 1.47$. b) Spectra of the two microphone signals for the $M_j = 1.47$ case of uniform-exit rectangular jets, yaw angle $\alpha = 10$ deg. Channel 2 is shifted arbitrarily above to show the peaks clearly. c) Cross-bicoherence spectra between the two microphone signals at $M_j = 1.47$.

The phase information revealed that in this case the jets ware antisymmetrically coupled. The phase plots between the microphone time-series signals also showed antisymmetric coupling for this case and a lot of unsteadiness for the Mach number 1.47 case, indicative of coupling transition.

2. *Single-Beveled Jets (B.A. = 10 Deg) in Contradirected Configuration*

Two yaw angles (10 and 20 deg) were considered for studying the nonlinear coupling behavior. It was observed that the 10-deg case showed rich nonlinear coupling compared to the 20-deg case. Both clustered and lineal interactions were present in each case. However, the 10-deg yaw case showed rich clustering activity compared with all other cases in this paper. The interaction density was very high owing to extreme clustering (see Fig. 9a). The 20-deg yaw case for the same Mach number showed lineal interactions with sparse clustering. An interesting observation is that whenever clustering is intense, several nonharmonically related frequencies are involved, bringing down the role of the screech frequency and its higher harmonics. This trend was observed in the 10-deg yaw case in which the role of screech frequency is minimal and those of nonharmonic

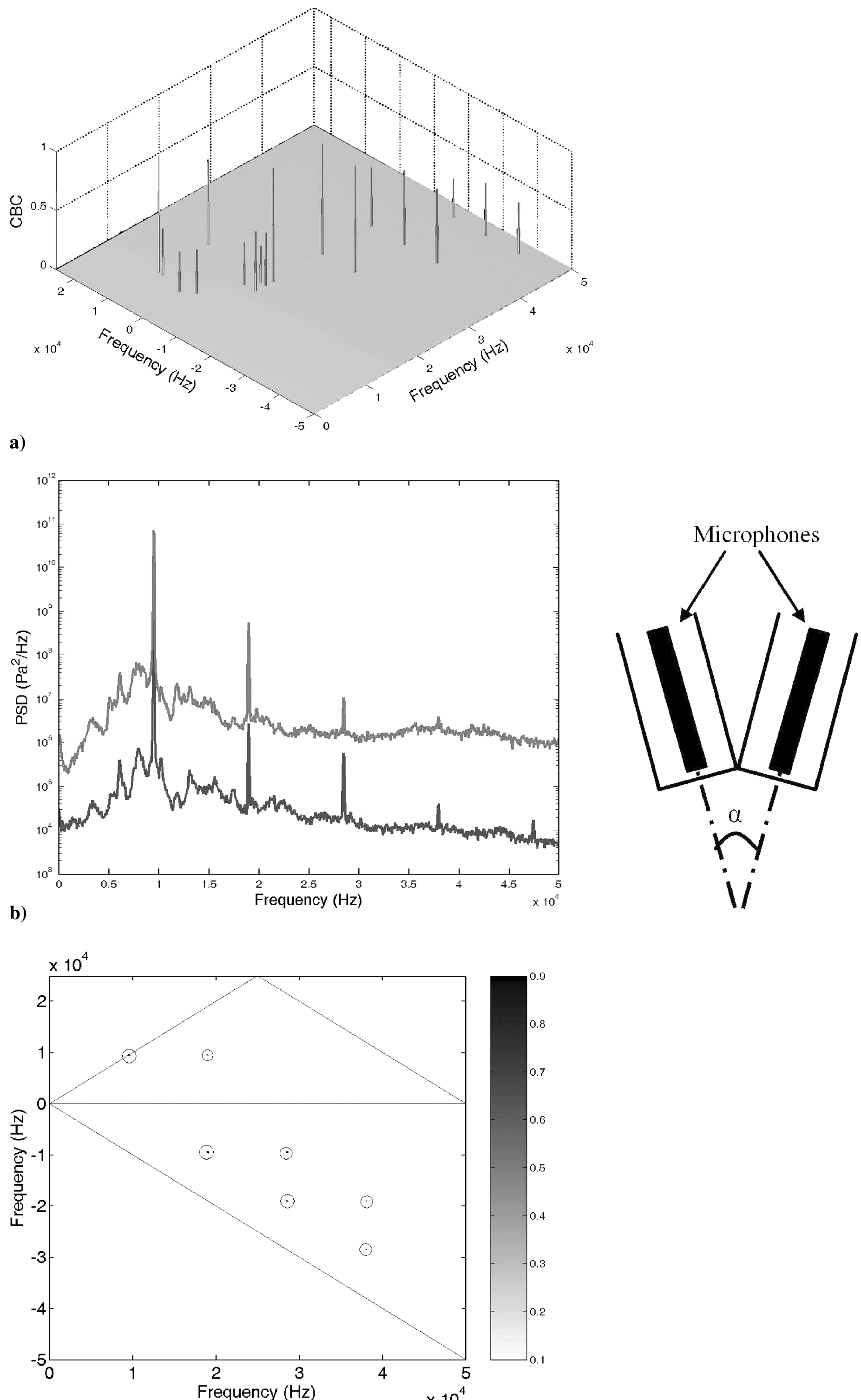


Fig. 8 Linear and CBC spectra for uniform-exit rectangular jet configurations at Mach number 1.53 as measured by microphones, located at the spanwise center of the nozzles as shown in the schematic: a) Three-dimensional view of the cross-bicoherence spectra between the two microphone signals at $M_j = 1.53$. b) Spectra of the two microphone signals for the $M_j = 1.53$ case of uniform-exit rectangular jets, yaw angle $\alpha = 10$ deg. Channel 2 is shifted arbitrarily above to show the peaks clearly. c) Cross-bicoherence spectra between the two microphone signals at $M_j = 1.53$.

frequencies are dominant. Lineal interactions dominate the 20-deg yaw case (see Fig. 9c), with the dominant role of screech frequency. Because the interaction densities for both these cases are large, we refrain from tabulating the cross-bicoherence values.

3. *Effect of Yaw Angle on Nonlinear Coupling*

Uniform-rectangular-nozzle configuration and 10-deg bevel arrowhead configuration were studied for their nonlinear coupling behavior. The cross-bicoherence spectra were obtained for the entire Mach-number range, and nonlinear metrics such as interaction density were calculated. It may be pointed out that the interaction density metric[17] refers to the number of nonlinear interactions present in the cross-bicoherence (CBC) spectra, above a certain threshold value of CBC. The threshold value in the present study is chosen as 0.4.

It was recognized that beveling, by itself, tends to vector away the jet owing to unequal pressure relief on the two sides of the nozzle

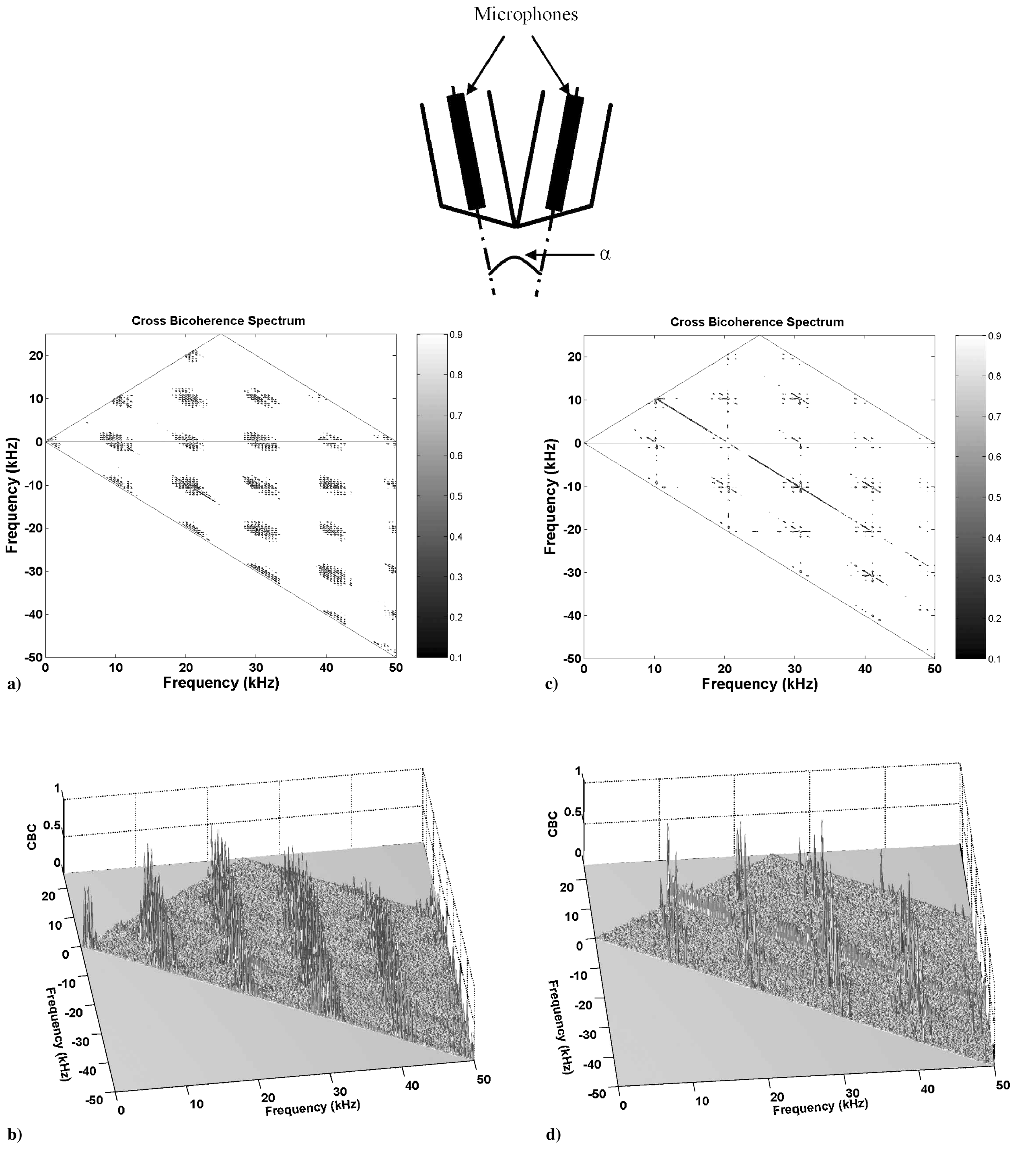


Fig. 9 Cross-bicoherence spectra for arrowhead ($B.A. = 10$ deg) configurations at Mach number 1.51 as measured by microphones, located at the spanwise center of the nozzles as shown in the schematic: a) Contour plot of cross-bicoherence for $\alpha = 10$ deg. b) Three-dimensional plot of the cross-bicoherence for $\alpha = 10$ deg. Note the intense clustering evident in a) and b). c) Contour plot of cross-bicoherence for $\alpha = 20$ deg. d) Three-dimensional plot of the cross-bicoherence for $\alpha = 20$ deg. Note the lineal/sparsely clustered for this yaw angle.

axis. Thus, although the beveled exit plays the role of an aerodynamic thrust-vectoring mechanism, yawing physically changes the axis of the jet. This poses an interesting question: What would be the combined effect of the two (yawing and beveling) in the case of arrowhead nozzles wherein bevel would tend to vector the jet away from its axis, which can be compensated for by yawing?

To answer this question, 10-deg bevel arrowhead nozzles, set at a yaw angle of 20 deg, which is geometrically similar to the uniform rectangular nozzles (β is 180 deg for both configurations), were studied. It may be recalled that the linear coupling behavior evidenced by the phase plots were identical for both these cases. If the nonlinear coupling behavior is the same, it must reflect in the cross-bicoherence metrics such as interaction density. Therefore, interaction densities for these two configurations have been plotted for all Mach numbers of the present study in Fig. 10. It is observed that the uniform rectangular jets show markedly lower interaction density compared to the arrowhead case. Thus, although the exits are made similar, the kinematic differences make the yawed arrowhead configuration more nonlinear compared to the uniform nozzles. This highlights the importance of nonlinear spectra in unearthing subtle differences in apparently similar flow situations.

Table 3 Cross-bicoherence values for various interactions for the Mach number 1.53 case

f_1, Hz	f_2, Hz	CBC	Resultant frequency, Hz	Remarks
9,491	9,452	0.9754	18,943	Self-interaction of fundamental freq.
18,982	−9,518	0.9570	9,464	Harmonic and fundamental
28,571	−9,518	0.9284	19,053	Second harmonic and fundamental
28,571	−19,003	0.8971	9,568	Second harmonic and first harmonic
19,080	9,452	0.7181	28,532	First harmonic and fundamental in a sum interaction (magnitude less than the diff. int.)
38,063	−19,003	0.6276	19,060	Interaction lower in hierarchy producing the first harmonic mode
38,063	−28,585	0.6275	9,478	Lower-order interaction producing the fundamental
15,656	−9,420	0.5002	6,236	
38,063	−9,420	0.4955	28,643	
17,515	−9,420	0.4440	8,095	
47,554	−28,488	0.4373	19,066	
47,554	−38,070	0.4314	9,484	
9,589	−1,499	0.3629	8,090	
15,656	−6,193	0.3537	9,463	
10,274	9,452	0.3495	19,726	
8,121	1,434	0.3370	9,555	
47,554	−19,003	0.3216	28,551	
17,515	−8,051	0.3096	9,464	
9,393	8,083	0.3038	17,476	

Table 4 Quantum of nonlinearities associated with frequencies identified in linear spectra

Spectral peak, Hz	Total CBC	Remarks
37,964	——	No nonlinearity identified.
9,497	3.913	Fundamental frequency
18,994	2.969	First harmonic
28,467	1.04	Second harmonic
6,128	0.5	Nonharmonically related frequency

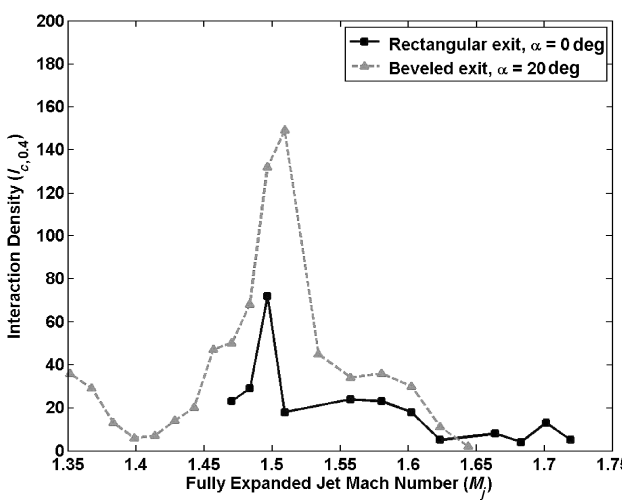


Fig. 10 Interaction density plots comparing the two twin-jet configurations with the same total angle, $\beta = 180$ deg. The legend indicates the configuration and the corresponding yaw angle α.

V. Conclusions

This paper focuses on the effects of nozzle orientation on the dynamic internozzle pressures for jets emanating from complex-geometry nozzles. Three configurations were considered in this study: uniform-exit rectangular nozzles, single-beveled nozzles with $B.A. = 30$ deg in the codirected configuration, and single-beveled nozzles with $B.A. = 10$ deg in the contradirected configuration. Experiments were conducted by changing the nozzle orientation by yawing the nozzles, and analyses were conducted using both linear and nonlinear techniques. The important conclusions that could be drawn from these analyses are summarized here.

Linear spectral analysis revealed the following:

1) For uniform-exit rectangular nozzles, as the yaw angle was increased the jets were vectored toward each other and the peak sound-pressure level in the internozzle region was reduced at all the fully expanded Mach numbers. At some of the fully expanded Mach numbers, reductions ranging from 15 dB up to 30 dB in the peak sound-pressure level were recorded, a trend corroborated by previous work.[14,15]

2) For uniform-exit rectangular nozzles, at no yaw, the jets were coupled in the antisymmetric mode at the lower fully expanded jet Mach numbers and symmetrically coupled at the higher fully expanded Mach numbers. When the jets were yawed, the symmetric coupling was no longer observed, and instead the jets were antisymmetrically coupled over the entire range of fully expanded jet Mach numbers.

3) For single-beveled nozzles, with $B.A. = 10$ deg, tested in the contradirected configuration, studying the phase map and the corresponding sound-pressure levels over the range of fully expanded Mach numbers revealed that symmetric coupling persisted at the higher Mach numbers. In spite of this, at certain yaw angles, the sound-pressure levels were lower than the case with no yaw. This is contrary to the existing notion that symmetric coupling increases the internozzle sound-pressure levels.

Nonlinear spectral analysis revealed the following:

1) For the uniform-exit rectangular nozzles, at the fully expanded Mach number where the coupling transitioned from antisymmetric to symmetric, the number of nonlinear interactions between nonharmonic frequencies was significantly greater than that for the fully expanded Mach number where the jets were antisymmetrically coupled.

2) For the single-beveled nozzles with $B.A. = 10$ deg tested in the contradirected configuration, the smaller yaw angles showed intense clusters of nonlinear interactions at a given fully expanded Mach number ($M_j = 1.51$) as compared to the interactions at the same fully expanded Mach number for greater yaw angles.

3) Analysis of geometrically similar configurations of uniform-exit rectangular nozzles and single-beveled nozzles leads us to conclude that the effect of yawing is to increase the amount of nonlinearities in the interaction between the jets, a result that cannot be arrived at by linear-analysis techniques alone.

Acknowledgment

This research was funded by the Air Force Office of Scientific Research (AFOSR) with John Schmisseur as program manager.

References

[1]Norum, T. D., "Screech Suppression in Supersonic Jets," *AIAA Journal*, Vol. 21, No. 2, 1983, pp. 235–240.

[2]Berndt, D. E., "Dynamic Pressure Fluctuations in the Internozzle Region of a Twin-Jet Nacelle," Society of Automotive Engineers, Paper 84-1540, Oct. 1984.

[3]Wlezien, R. W., "Nozzle Geometry Effects on Supersonic Jet Interaction," *AIAA Journal*, Vol. 27, No. 12, 1989, pp. 1361–1367; also AIAA Paper 87-2694, Oct. 1987.

[4]Tam, C. K. W., and Seiner, J. M., "Analysis of Twin Supersonic Plume Resonance," AIAA Paper 87-2695, Oct. 1987.

[5]Seiner, J. M., Manning, J. C., and Ponton, M. K., "Dynamic Pressure Loads Associated with Twin Supersonic Plume Resonance," *AIAA Journal*, Vol. 26, No. 8, 1988, pp. 954–960.

[6]Morris, P. J., "Instability Waves in Twin Supersonic Jets," *Journal of Fluid Mechanics*, Vol. 220, 1990, pp. 293–307.

[7]Tam, C. K. W., Shen, H., and Raman, G., "Screech Tones of Supersonic Jets from Beveled Rectangular Nozzles," AIAA Paper 97-0143, Jan. 1997.

[8]Rice, E. J., "Jet Mixer Noise Suppressor Using Acoustic Feedback," U.S. Patents 5,325,661 and 5,392,597, 1995.

[9]Raman, G., "Screech Tones from Rectangular Nozzles with Spanwise Oblique Shock-Cell Structures," *Journal of Fluid Mechanics*, Vol. 330, 1997, pp. 141–168.

[10]Raman, G., and Taghavi, R., "Coupling of Twin Rectangular Supersonic Jets," *Journal of Fluid Mechanics*, Vol. 354, 1998, pp. 123–146.

[11]Raman, G., "Coupling of Twin Supersonic Jets of Complex Geometry," *Journal of Aircraft*, Vol. 36, No. 5, 1999, pp. 743–749.

[12]Viswanathan, K., "Nozzle Shaping for Reduction of Jet Noise from Single Jets," AIAA Paper 2004-2974, May 2004.

[13]Shaw, L., "Twin Jet Screech Suppression," *Journal of Aircraft*, Vol. 27, No. 8, 1990, pp. 708–715.

[14]Zilz, D. E., "Near-Field Acoustics of Supersonic Jets from Twin Two-Dimensional Nozzles," McDonnell Douglas Aircraft Company, St. Louis, MO, July 1989.

[15]Zilz, D. E., and Wlezien, R. W., "The Sensitivity of Near-Field Acoustics to the Orientation of Twin Two-Dimensional Supersonic Nozzles," AIAA Paper 90-2149, July 1990.

[16]Panickar, P., Srinivasan, K., and Raman, G., "Aeroacoustic Features of Coupled Twin Jets with Spanwise Oblique Shock-Cells," *Journal of Sound and Vibration*, Vol. 278, No. 1–2, 2004, pp. 155–179.

[17]Srinivasan, K., Panickar, P., Raman, G., Byung-Hun, K., and Williams, D., "Study of Supersonic Twin Jet Coupling Using Higher Order Spectral Analysis," AIAA Paper 2003-3871, June 2003.

[18]Ponton, M. K., and Seiner, J. M., "Acoustic Study of B Helical Mode for Choked Axisymmetric Nozzle," *AIAA Journal*, Vol. 33, No. 3, 1995, pp. 413–420.

[19]Walker, S. H., and Thomas, F. O., "Experiments Characterizing Nonlinear Shear Layer Dynamics in a Supersonic Rectangular Jet Undergoing Screech," *Physics of Fluids*, Vol. 9, No. 9, 1997, pp. 2562–2579.